Predictive Modeling for Parametric Analysis of an Air Conditioning System in a 400-seat Auditorium in Warri, Sub-tropical Nigeria

E. D. Edafiadhe a, C. C. NwobiOkoye b, K. E. Madu b and J. L. Chukwuneke c*

a Department of Mechanical Engineering, Delta State University of Science & Technology, Ozoro, Nigeria.
b Department of Mechanical Engineering, Chukwuemeka Odumegwu Ojukwu University, Uli, Nigeria.
c Department of Mechanical Engineering, Nnamdi Azikiwe University, Awka, Nigeria.

Authors’ contributions
This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information
DOI: 10.9734/JERR/2022/v23i517611

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/91132

Received 16 June 2022
Accepted 24 August 2022
Published 10 September 2022

ABSTRACT

Considering the fact that Warri is located in the sub-tropical region of Nigeria which is characterized by excessively high temperature and high humidity, it is necessary to develop an air-conditioning system which can reduce the hot and ambient temperature within a confined space for human comfort. This research presents the design of an air-conditioning system for comfort of occupants in a 400-Seater auditorium. The three independent variables considered for this design are; the number of occupants, air exchange rate and the outdoor temperature. The auditorium was properly measured and the heat generated by the bulbs, appliances and occupants were adequately considered to get the required cooling load. Regression analysis was used to predict and determine the cooling load. Optimization analysis was also done to determine the optimal condition for maximum cooling load. The plant capacity that will provide thermal comfort for the 400 occupants was calculated to be 25 ton of refrigeration (TR).

Keywords: Air-conditioning system; parametric analysis; predictive model; optimization; temperature; heat.

*Corresponding author: Email: jl.chukwuneke@unizik.edu.ng;
ABBREVIATIONS

ACH : Air changes per hour;
DBT : Dry bulb temperature;
WBT : Wet bulb temperature;
TR : Ton of refrigeration;
RH : Relative humidity;
PRS : Polynomial response surface;
DOE : Design of experiment;
CCD : Central composite design;
GA : Generic algorithm;
TCL : Total cooling load;
TSH : Total sensible heat;
TLH : Total latent heat.

1. INTRODUCTION

Air conditioning can be explained as any process that alters the humidity and temperature of indoor air to favorable conditions, typically with the aim of distributing the conditioned air to a space that is occupied in order to improve comfort. Air conditioning is any type of technology used for cooling, purifying, humidifying, heating, ventilating, de-humidifying or moving air, modifying its temperature [1,2,3].

In general, an air conditioner is a mechanical device that lowers the air temperature. The cooling is mostly done through a simple refrigeration cycle, but sometimes by evaporation, commonly for the cooling of buildings and motor vehicles (Jadhav & Lele, 2015); [4,5,6]. Air conditioning can be provided also by the simple process called free cooling which uses pumps to circulate a coolant from a cold source, which in turn acts as a heat sink for the energy that is removed from the cooled space [7-9].

Generally, air conditioners can be classified into window unit air-conditioners, central unit air-conditioner, and split unit air-conditioner [10,11,12]. The various components of a window unit air conditioner, e.g. condenser, expansion valve, compressor, evaporator and cooling coil are designed as a single system unit. This type of air conditioning unit is usually placed in a wall opening whereby interior air is cooled as a fan blows it over the evaporator. The heat extracted from the inside is discharged to the outside environment as the second fan outside blows outdoor air over the air condition system condenser. For large buildings with several rooms, each room can be made to have a cooling unit allowing cooling of each room separately.

The central unit conditioner is preferred for cooling big residential houses, offices, hotels, cinema halls, factories, etc. [8]. The central unit air conditioner is made of huge compressor that has the capacity to produce tons of conditioned air [9]. Split unit air conditioning system comprises two parts; the outdoor unit and indoor unit. The outdoor unit fitted outside a room houses components like the compressor, condenser, and the expansion valve [6,13]. The indoor unit comprises the evaporator, and cooling fan [13,14]. Considering that Warri is located in the sub-tropical region of Nigeria characterized by excessive temperature and high humidity [15], it became necessary to design an air conditioning system which can help reduce the hot ambient temperature within a confined space for human comfort. In this research work, the parametric analysis of an air conditioning system for a 400 seat capacity auditorium in Warri was carried out.

2. METHODOLOGY

2.1 Problem Description/Assumptions

The air conditioned auditorium is to be maintained at comfort condition of 22°C DBT and 50 – 55% R.H. The number of occupants and the ambient condition varies in the following order:

- Number of occupants: 400 persons
- Outdoor Temperature: (29 – 41°C) DBT and (25 - 32°C) WBT
- Air exchange rate: 0.46 – 0.54 [ACH]

The ventilation requirement inside the auditorium is such that 55% of the return air is re-circulated and mixed with 45% of the make-up air after the cooling coil. The condition of air leaving the cooling coil is to be maintained at 14 °C. Seating capacity of the auditorium is 400 persons.

Sensible heat given per person and the latent heat gain per person are assumed to be 320 kJ/hr and 100 kJ/hr respectively. The inside condition varies in accordance with the degree of activity of the occupants and intended use of the auditorium.

The following response functions can be considered: total cooling load (Watt); cooling coil capacity (TR); and amount of water removed by dehumidification (kg/s).

The system is represented in block diagram as Fig. 1. The block diagram description of the problem is given in Fig. 1(a) followed by an
equivalent representation on the psychrometric chart in Fig. 1(b).

Given that 55% of air from the auditorium is re-circulated and mixed with 45% of make-up air after the cooling coil, the mixing condition of air is shown by point 3 such that:

\[
\frac{\text{Length } 2 - 3}{\text{Length } 2 - 4} = \frac{55}{100} = 0.55
\]

Therefore, \( \text{Length } 2 - 3 = 0.55 \times (\text{Length } 2 - 4) \) (1)

Point 3 equally describes the condition of air entering the auditorium.

From material and energy balance analysis the total cooling load is given by;

\[
\text{Total cooling load} = \dot{m}_a (h_4 - h_3)
\] (2)

Where \( \dot{m}_a \) mass flow rate of supply air and \( h \) is is enthalpy.

Since make-up air is 45% of supply air, the mass flow rate of make-up air is given as; \( 0.45 \times \dot{m}_a \).

Outdoor design condition was considered as the observed range of outdoor dry bulb temperature in Petroleum Training Institute environment during 2020 summer season, given as \( 29 - 41^\circ C \) by the metrological report [16].

Infiltration into the auditorium is given as;

\[
Q_s = 1.2 Q \Delta T
\] (3)

Where \( Q \) is volumetric flow rate \( (m^3/s) \), \( Q_s \) is sensible heat \((W)\), \( \Delta T \) is the temperature difference between the outdoor and inside temperature \( (T_o - T_i) \) °C.

Size of the auditorium = \( (L \times B \times H) \) m³ (4)

Where \( L, B, H \) are the length, width and height of the auditorium respectively.

---

Fig. 1(a). Block diagram representation of the auditorium air-conditioning problem

Fig. 1(b). Representation of the auditorium air-conditioning problem on psychrometric chart
2.2 Design of Experiment and Response Prediction Via Polynomial Response Surface (PRS) Method

The important system variables that affect the total cooling load of an air-condition system include; number/activities of the occupants \((X_j)\), outdoor temperature \((X_2)\) and air exchange rate \((X_3)\). In a typical design it is necessary to optimize these variables to ensure that optimum plant capacity is achieved. This requires development of a suitable functional approximation model from which the overall behavior of the system could be studied. In this regard the polynomial response surface (PRS) method is proposed for statistical analysis of the system. The PRS is widely applied in reaction engineering, process design and general design situation to monitor quality of engineering products using mathematical principle. The PRS does not rely on high volume of data to yield accurate result. However, certain minimum number of data points is required. Thus, the implementation of PRS procedure utilized a design of experiment (DOE) developed for a range of the input variables called the central composite design (CCD). The design optimization study is intended to reveal the optimal plant capacity required to achieve the desired comfort of the occupants of the auditorium. The three key input variables were considered at the levels described in Table 1. The CCD template is presented in Table 2. Both the experimental design and the statistical analysis were implemented on model-based calibration (MBC3.3) toolbox found in MATLAB (R2007b). The modeling utilized a quadratic model of the generalized form presented in equation 1.

\[
f(X_jX_l) = \beta_0 + \sum_{j=1}^{k} \beta_j X_j + \sum_{l=1}^{k} \beta_l X_l + \sum_{i<j} \beta_{ij} X_jX_l + \epsilon \quad (5)
\]

Where, \(f(X_jX_l)\) is the objective function, \(\beta_0\) is the model intercept, \(k\) is the number of design variables. \(X_j, X_l \) and \(X_jX_l\) represent the first-order term(s), the second-order term(s), the interaction term(s) of the model, respectively. \(\beta_j, \beta_{ij}\) and \(\epsilon\) are the respective coefficients of the model terms. \(\epsilon\) is the random error of prediction.

<table>
<thead>
<tr>
<th>S/N</th>
<th>(X_1)</th>
<th>(X_2) °C</th>
<th>(X_3)</th>
<th>(Y(W))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(Y_1)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(Y_2)</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(Y_3)</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>(-\infty)</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>(\infty)</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>(-\infty)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>(\infty)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>0</td>
<td>(-\infty)</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>0</td>
<td>(\infty)</td>
<td>(Y_N)</td>
</tr>
</tbody>
</table>
2.3 Design Optimization Via Genetic Algorithm

The genetic algorithm (GA) is an evolutionary method for solving both constrained and unconstrained optimization problems that is based on the principle of natural selection. The algorithm starts with an initial population representing nominal individual solution. The genetic algorithm repeatedly modifies the population. At each step, the GA selects individuals at random from the current population to be parents and uses them to produce offspring for the next generation following three major genetic operators including: Selection, Crossover and Mutation.

Selection rules select the individuals, called parents that contribute to the population at the next generation based on the resulting fitness value. Crossover rules combine two parents to form children for the next generation. Mutation rules apply random changes to individual parents to form children. Over successive generations, the population “evolves” toward an optimal solution. GA can be applied to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear. The implementation and result using the genetic algorithm will be presented and discussed in section 3.

3. RESULTS AND DISCUSSION

3.1 Experimental Design and Statistical Modelling

The study basically involves calculation of the total cooling load (TCL) of the auditorium at operating condition as presented in Table 3. To this end, the contributions to the total sensible heat (TSH)/the total latent heat (TLH) resulting from human activities, the available facilities and infiltrations, calculated at various operating conditions prescribed by the experimental design were added up to obtain the calculated TCL, \( Y(W) \).

The information obtained for the observed eighteen data points were clearly not sufficient to describe completely the behavior of the system, thus further statistical analysis via mathematical model was conducted. The quadratic model presented in equation 5 was fitted to the input – output data set. Following the least squares approximation method, the unknown constant and coefficient terms of the fitting function were determined to obtain the actual predictive model for the total cooling load presented as equation 6. The total cooling load was also evaluated via the predictive model at the DOE test conditions and the results (Predicted Y) were juxtaposed with the calculated TCL (Y) in Table 3.

\[
Y(X_1^2, X_2^2, X_3^2) = 32485.2373 + 116.66X_1 - 22914.58X_3 + 1041.67X_2X_3 + 0.0000118X_2^2
\]  

(6)

Table 3. Calculated and predicted cooling load at some selected design points using the CCD

<table>
<thead>
<tr>
<th>S/N</th>
<th>( X_1 )</th>
<th>( X_2 ) °C</th>
<th>( X_3 )</th>
<th>( Y(W) )</th>
<th>Predicted ( Y'(W) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>35</td>
<td>0.5</td>
<td>62590</td>
<td>62590.21</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>35</td>
<td>0.5</td>
<td>62590</td>
<td>62590.21</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>35</td>
<td>0.5</td>
<td>62590</td>
<td>62590.21</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>35</td>
<td>0.5</td>
<td>62590</td>
<td>62590.21</td>
</tr>
<tr>
<td>5</td>
<td>81</td>
<td>31.4</td>
<td>0.48</td>
<td>46636</td>
<td>46636.03</td>
</tr>
<tr>
<td>6</td>
<td>319</td>
<td>31.4</td>
<td>0.48</td>
<td>74403</td>
<td>74402.97</td>
</tr>
<tr>
<td>7</td>
<td>81</td>
<td>38.6</td>
<td>0.48</td>
<td>50236</td>
<td>50236.03</td>
</tr>
<tr>
<td>8</td>
<td>319</td>
<td>38.6</td>
<td>0.48</td>
<td>78003</td>
<td>78002.97</td>
</tr>
<tr>
<td>9</td>
<td>81</td>
<td>31.4</td>
<td>0.52</td>
<td>47028</td>
<td>47027.78</td>
</tr>
<tr>
<td>10</td>
<td>319</td>
<td>31.4</td>
<td>0.52</td>
<td>74795</td>
<td>74794.72</td>
</tr>
<tr>
<td>11</td>
<td>81</td>
<td>38.6</td>
<td>0.52</td>
<td>50928</td>
<td>50927.78</td>
</tr>
<tr>
<td>12</td>
<td>319</td>
<td>38.6</td>
<td>0.52</td>
<td>78695</td>
<td>78694.72</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>35</td>
<td>0.5</td>
<td>39257</td>
<td>39257.11</td>
</tr>
<tr>
<td>14</td>
<td>400</td>
<td>35</td>
<td>0.5</td>
<td>85924</td>
<td>85924.25</td>
</tr>
<tr>
<td>15</td>
<td>200</td>
<td>29</td>
<td>0.5</td>
<td>59465</td>
<td>59465.21</td>
</tr>
<tr>
<td>16</td>
<td>200</td>
<td>41</td>
<td>0.5</td>
<td>65715</td>
<td>65715.21</td>
</tr>
<tr>
<td>17</td>
<td>200</td>
<td>35</td>
<td>0.46</td>
<td>62049</td>
<td>62048.46</td>
</tr>
<tr>
<td>18</td>
<td>200</td>
<td>35</td>
<td>0.54</td>
<td>63132</td>
<td>63131.96</td>
</tr>
</tbody>
</table>
The performance of the resulting model was first evaluated using the resulting summary of statistics (SOS) for the model presented in Table 4. The SOS reflects high prediction accuracy of the model. The $R^2$, the adjusted $R^2$ and the PRESS $R^2$ all stand at maximum value of 1. These are indications that 100% of the overall system behavior can be predicted via the predictive model within the range of input variables. The result of maximum value of 1 is valid in this case since the original response data were obtained by calculation using exiting theoretical relationships and not from experimentation. Thus, random error usually associated with experimental irregularities was totally avoided. The RMSE value is sufficiently close to zero. This also suggests that the prediction error was adequately minimized. Moreover, there is no indication of over-fitting of the model. The high prediction accuracy of the quadratic model is further illustrated by the agreement witnessed between the predicted response and the calculated response over the entire design space as recorded in both Table 3 and Fig. 2.

### 3.2 Interaction Effects of the Design Variables

Looking at the results holistically an overall linear interaction were observed. For various significant interactions presented, two factors were varied simultaneously while the third factor is kept at its mean value. The combined effects of the number of occupants and outdoor temperature on the predicted total cooling load are presented in Fig. 3. From the results, the number of occupants appears to be the most influential factor for the studied range of parameters. The TCL was found to increase progressively along a straight line while both the number of occupants and the outdoor temperature were increased at mean air exchange rate. This observation agrees with the intuition that higher cooling efforts (or greater plant capacity) will be required to maintain cooling comfort for increased number of occupants or increased outdoor temperature.

Similar results were recorded with respect to the interactive effects of the outdoor temperature and the natural air exchange rate as shown in Fig. 4. However, the outdoor temperature condition show relatively greater effect on TCL compared to air exchange rate. The overall result suggests that maximum TCL could be obtained towards the positive axial points of the three design variables.

### 3.3 Prediction of Optimal Cooling Load Via Genetic Algorithm (GA)

In order to specify the appropriate plant capacity that would guarantee occupants’ comfort in the auditorium over a wide range of ambient condition and human activities, the optimal cooling load must be evaluated. Thus, maximization of the total cooling load, $Y$ was considered a major design objective. The response $Y$ was identified as a function of three independent variables including; the number of occupants $X_1$, the outdoor temperature $X_2$ and the air exchange rate $X_3$ which are considered the

![Fig. 2. Comparison of the calculated and predicted response](image-url)
The optimization study was implemented in global optimization toolbox found in MATLAB. The steps involved include: selection of optimization algorithm, calling of the fitting function via the function handle, setting of the number of variables, specification of the boundary points and the algorithm specific parameters.

The GA specific optimal parameters utilized in the study are: crossover fraction ‘0.9’; rossover function scattered; mutation function ‘adaptive’; population size ‘100’; engeration ‘100’.

The predicted optimal solution is presented in Table 5.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$ °C</th>
<th>$X_1$ (ACH)</th>
<th>$Y$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>390</td>
<td>38.2</td>
<td>0.52</td>
<td>86851.5</td>
</tr>
</tbody>
</table>

Fig. 3. Combined effects of the number of occupants and the outdoor temperature on the predicted total cooling load

Fig. 4. Combined effect of the outdoor temperature and the air exchange rate on the predicted total cooling load
From the predicted optimal cooling load, $Y_{\text{optimal}}$, the optimal plant capacity in tons of refrigeration is given by:

$$\frac{Y_{\text{optimal}}}{3500} = \frac{86851.5}{3500} = 24.817\text{TR}$$

(7)

Considering reasonable design tolerance, the installed capacity of the air conditioning plant should be 25TR.

4. CONCLUSION

The design of an air-conditioning unit for a 400 seat capacity auditorium was successfully carried out. A plant of 25TR capacity was recommended for the auditorium to guarantee occupants comfort over a wide range of operating conditions. Although the number of occupants appears to be the most important variable, the two other identified system variables including; outdoor temperature and natural air exchange rate equally have considerable effects on the total cooling load of the auditorium. A model equation for predicting the total cooling load for the proposed auditorium air conditioning project at typical design condition was developed and the optimal plant capacity for the proposed auditorium air conditioning project was determined. In terms of performance, the optimal value of 24.81TR indicates an excellent design system.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES


© 2022 Edafiadhe et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here:
https://www.sdiarticle5.com/review-history/91132