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Novel Characterizations of the JK Bistables (Flip Flops)

Ali Muhammad Ali Rushdi[1*] and Fares Ahmad Muhammad Ghaleb[1]

[1]Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, P.O.Box 80204, Jeddah, 21589, Saudi Arabia.

Authors' contributions

This work was carried out in collaboration between the two authors. Author AMAR envisioned and designed the study, performed the symbolic analysis, constructed the variable-entered maps and wrote the entire manuscript. Author FAMG verified the mathematical derivations, managed the literature search and drew the various figures. Both authors read and approved the final manuscript.

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ABSTRACT

The JK flip flop is a flexible type of bistable elements that has extensive uses in digital electronics and control circuits. It is usually described by its characteristic equation or next-state table (used for analysis purposes) and its excitation table (used for synthesis purposes). This paper explores a variety of novel characterizations of JK flip flops. First, equational and implicational descriptions are presented, and methods of logic deduction are utilized to produce complete statements of all propositions that are true for a general JK flip flop. Next, methods of Boolean-equation solving are employed to find all possible ways to express the excitations in terms of the present state and next state. The concept of Boolean quotient is used to impose certain requirements so as to find particularly useful expressions of the excitations. Relations of JK flip flops to other types of flip flops are also explored. This paper is expected to provide an immediate pedagogical benefit, and to help facilitate the analysis and synthesis of sequential digital circuits.

Keywords: Flip flop; characterization; logic deduction; Boolean-equation solving; Boolean quotient; excitation.

**Corresponding author: Email: alirushdi@gmail.com, arushdi@kau.edu.sa, arushdi@ieee.org, arushdi@yahoo.com;*

1. INTRODUCTION

Digital circuits are broadly divided into combinational (combinatorial) and sequential circuits [1-6]. A combinational circuit is sometimes labelled as time-independent in the sense that its current outputs depend only on its current inputs, *i.e.*, previous (past) inputs for a combinational circuit have no effect on its present outputs. A combinational circuit also does not require any clocks, since synchronization is not an issue for its operation. By contrast, a sequential circuit has “memory”; *i.e.,* a capability to remember previous inputs, so that its present outputs depend not only on its present inputs but also on the sequence of its past inputs. Such a dependence on input history is manifested or summarized as a “state” of the circuit. So, instead of saying that the present outputs of a sequential circuit are functions of its entire input history, it might suffice to say that these outputs are functions of the present input and present state. Therefore, a sequential circuit is also called a finite-state machine (FSM) or a finite-state automaton (FSA) when it has a finite number of states. Note that the concepts of “memory” and “state,” which are at the heart of the definition of a sequential circuit are totally missing from the description of a combinational circuit. Though sequential circuits might be asynchronous, they are dominantly clocked or synchronous. In a synchronous sequential circuit, an electronic oscillator called a clock generates a sequence of repetitive pulses which is distributed to all the memory or storage elements in the circuit. These elementary memory cells are called flip flops, or one-bit registers (albeit historically known as bistables or bistable multivibrators).

A flip-flop is a circuit that has two stable states and can be used to store a single bit (binary digit) of data; with one of the two states of the flip flop representing a "one" and the other representing a "zero". Flip flops are (together with combinational gates) the basic building blocks of a sequential circuit, with the state of the sequential circuit being (acquired and) determined by the states of the flip flops it contains. A flip flop is usually named in terms of its inputs or excitations. There are many well-known types of flip flops, including those with a single input (such as the D flip flop and the T flip flop), those with two inputs (such as the SR flip flop, the JK flip flop, and the DE flip flop), one with three inputs (namely the RST flip flop), and one with five inputs (namely the $FIVEX$ flip flop). Out of the types mentioned, the SR flip flop and the RST flip flop have constraints on their excitations, while the rest enjoy the merit of unconstrained operation.

This paper is a tutorial exposition about flip flops in general, and about JK flip flops, in particular, where we use the term “JK flip flop” in its restrictive sense of “clocked master-slave JK flip flop”. After surveying conventional characterizations of JK flip flops, we employ novel mathematical methods for further characterization of them. The methods used include those of logic deduction, Boolean-equation solving, and don't-care assignment. We devote most of our current exposition to the JK flip-flop, which is a very versatile device, and is probably the most commonly used form of flip flop in digital electronics and in control circuits. Besides the merit of unconstrained operation, the JK flip-flop enjoys two distinctive advantages:

- It possesses the minimum number of excitations that can control its next-state variable to take any of the four possible values taken by a Boolean function of a single variable representing the present-state value, namely: the present state itself, its complement, logic 0, and logic 1.
- Its excitations can be made independent of its present state. There is no need for some external feedback from its outputs to its inputs, thanks to the existence of such feedback internally within the circuit structure of the flip flop itself.

This paper is a prelude for some work on the efficient design of synchronous sequential circuits, based on a novel state diagram [7,8] and the utilization of variable-entered Karnaugh maps [9-13]. The paper serves the same theme covered by earlier contributions on flip-flop equations [14-21], albeit with a different stress and a novel methodology. It is excluded to binary crisp flip flops, as opposed to multi-valued [22] or fuzzy [23-25] ones. Its tutorial nature might make it a useful addition to the pedagogical literature on switching theory and digital design, thereby enhancing students’ understanding, sharpening their skills of problem solving, and (possibly) remedying their misconceptions [26-31].

The organization of the remainder of this paper is as follows. Section 2 discusses conventional characterization methods for the JK flip flop including the verbal, tabular, graphical, and algebraic methods. Section 3 uses the Modern Syllogistic Method (MSM) of logic deduction to ferret out all that can be said about the JK flip flop in the most compact form. Section 4 applies methods of 'big' Boolean-equation solving to find all possible solutions of the excitations in terms of the present and next states. Section 5 explores possibilities for imposing certain requirements on the aforementioned solutions. Section 6 investigates the relations among standard types of flip flops. Section 7 concludes the paper. To make the paper self-contained, it is supplemented with an appendix on the “Boolean Quotient,” a crucial concept for the derivations in Sec. 5.

2. CONVENTIONAL CHARACTERIZATIONS OF THE JK FLIP FLOP

In this paper, we use the term JK flip flop (bistable) to refer to a master-slave clocked flip flop. This flip flop uses a pair of flip flops so as to eliminate hazards. It works reliably regardless of how long a clock pulse or signal 1 at input Jor input K lasts. Since the reading of information into the flip flop and the establishment of new output values are done at different times, the outputs of the flip flop can be completely prevented from feeding back to the inputs while the circuit containing the flip flop is still in transition.

A *verbal* characterization of the JK flip flop is achieved by the following statements, which describe its operation after each clock pulse:

- ✓ When the input $J = 1$ and the input $K = 0$, the next-state output Y_i is set to 1.
- ✓ When the input $J = 0$ and the input $K = 1$, the next-state output Y_i is reset to 0.
- ✓ When the inputs J and K are both equal to 0 , the next-state output Y_i retains its present-state value y_i.
- ✓ When the inputs J and K are both equal to 1 , the next-state output Y_i toggles or switches to the complementary value $\bar{y}_i$ of its present-state value.

There is a plethora of other conventional methods for characterizing a JK flip flop [1-6]. These include *tabular* methods such as the expanded next-state table (Table 1(a)), the reduced (variable-entered) next-state table (Table 1(b)), and the excitation table (Table 1(c)). Characterizing methods can also be of a *graphical* nature such as the expanded next-state Karnaugh map (Fig. 1(a)), the reduced (variable-entered) next-state Karnaugh map (Fig. 1(b)), the expanded transition Karnaugh map (Fig. 1(c)), the state diagram (Fig. 1(d)), the expanded excitation Karnaugh maps (Fig. 1(e)), the reduced (variable-entered) Karnaugh map (Fig. 1(f)), and the excitation Karnaugh maps in terms of four-valued transitions (Fig. 1(g)). In both Table 1 and Fig. 1, we use the symbols y_i, Y_i , J_i , and K_i to denote the present and next states of flip flop number i and the two excitations of this flip flop, respectively. The symbol δy_i represents a four-valued (rather than binary) variable defined as $\delta y_i = 0$ when $Y_i = y_i = 0$, $\delta y_i = 1$ when $Y_i = y_i = 1$, $\delta y_i = \Delta$ when the flip-flop state goes *up* from $0\ to\ 1$, and $\delta y_i = \nabla$ when the flip-flop state goes *down* from $1\ to\ 0$ [21], *i.e.*,δy_i is given by (See Table 1(c))

$$\delta y_i = (0)\bar{y}_i\bar{Y}_i \vee (\Delta)\bar{y}_i Y_i \vee (\nabla) y_i \bar{Y}_i \vee (1) y_i Y_i. \quad (1)$$

The flip flop might also be characterized *algebraically*. Its algebraic characteristic equation (next-state equation)

$$Y_i = J_i \bar{y}_i \vee \bar{K}_i\, y_i, \quad (2)$$

might be readily deduced from either Fig. 1(a) or Fig. 1(b). Note that equation (2) is a minimal-sum representation [2,20] for the next-state Y_i . A complete-sum representation [2,20] of Y_i is

$$Y_i = J_i\, \bar{y}_i \vee \bar{K}_i\, y_i \vee J_i\, \bar{K}_i, \quad (3)$$

where the consensus $J_i\bar{K}_i$ of the original implicants $J_i\bar{y}_i$ and $\bar{K}_i y_i$(with respect to the biform variable y_i) is added to (2) to obtain (3).

Table 1. Conventional tabular descriptions of the JK flip flop

J_i	K_i	y_i	Y_i	δy_i
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	∇
1	0	0	1	Δ
1	0	1	1	1
1	1	0	1	Δ
1	1	1	0	∇

a) Expanded next-state table

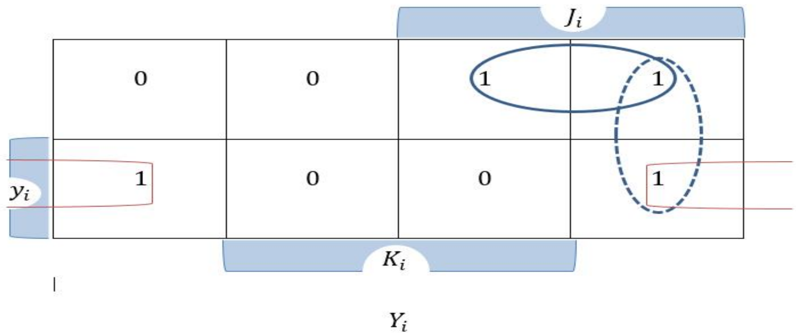
J_i	K_i	Y_i	δy_i
0	0	y_i	$0/1$
0	1	0	$0/\nabla$
1	0	1	$\Delta/1$
1	1	$\bar{y}_i$	Δ/∇

b) Reduced (variable-entered) next-state table

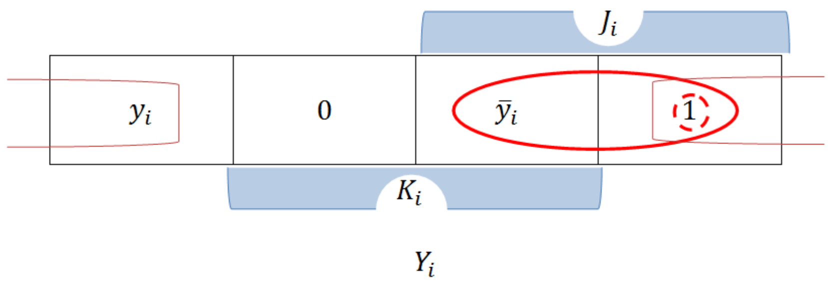
y_i	Y_i	δy_i	J_i	K_i
0	0	0	0	d
0	1	Δ	1	d
1	0	∇	d	1
1	1	1	d	0

c) Excitation table

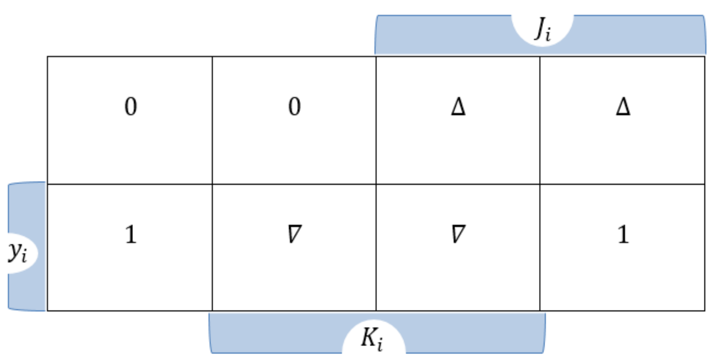
By contrast to certain other types of flip flops (such as the SR flip flop or the RST flip flop), the characteristic equation (2) of the JK flip flop is not subject to any constraint.



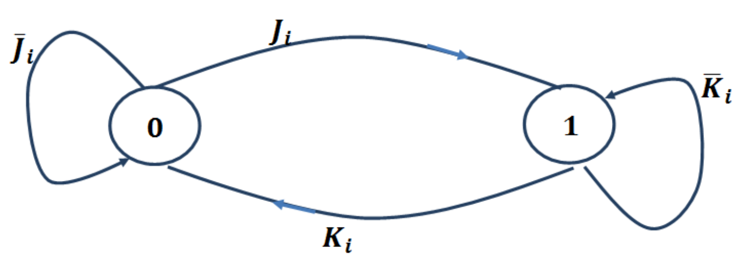
a) Expanded next-state Karnaugh map



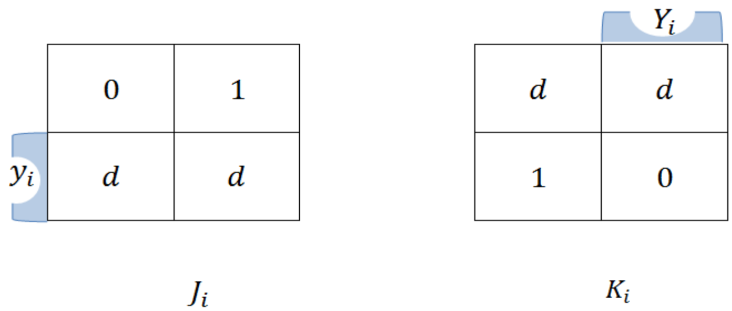
b) Reduced (variable-entered) next-state Karnaugh map



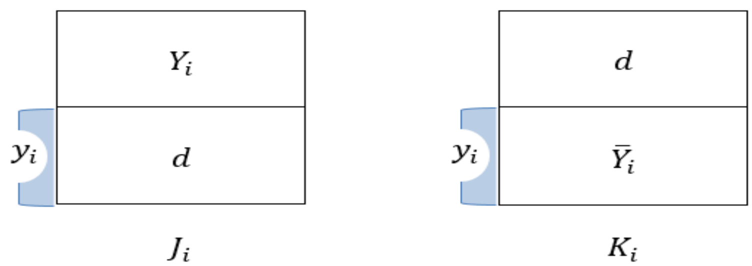
c) Expanded transition Karnaugh map



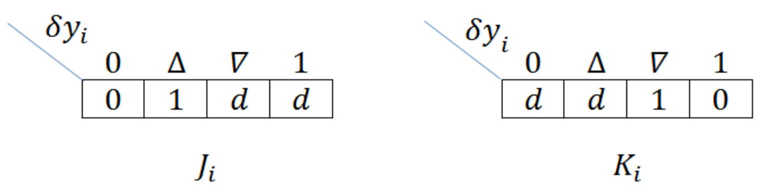
d) State diagram



e) Expanded excitation Karnaugh maps



f) Reduced (variable-entered) excitation Karnaugh map



g) Excitation Karnaugh maps in terms of four valued transition

Fig. 1. Conventional map descriptions of the JK flip flops

3. EQUATIONAL AND IMPLICATIONAL DESCRIPTIONS

In this section, we use the Modern Syllogistic Method (MSM) [13,20,32-38] to ferret out from the characteristic equation (2) (viewed as a premise) all that can be concluded from it, with the resulting conclusions cast in the simplest or most compact form. First, we reformulate (2) as an equation whose *R.H.S.* is 0. *i.e.*,

$$f = f(Y_i.\ y_i.\ J_i.K_i) = 0, \tag{4}$$

where

$$\begin{aligned} f = Y_i \oplus (J_i\bar{y}_i \vee \bar{k}_i y_i) &= Y_i\overline{(J_i\bar{y}_i \vee \bar{K}_i y_i)} \vee \bar{Y}_i(J_i\bar{y}_i \vee \bar{K}_i y_i) \\ &= Y_i(\bar{J}_i\bar{y}_i \vee K_i y_i) \\ &\vee \bar{Y}_i(J_i\bar{y}_i \vee \bar{K}_i y_i). \end{aligned} \tag{5}$$

Then, we replace f in (5) by its complete-sum form using the Blake-Tison Method [33-38]. We note that (5) involves four variables, which are all biform. However, there is no consensus *w.r.t.* the three variables Y_i, J_i, andK_i. However, there are two consensuses $Y_i\bar{J}_iK_i$ and $\bar{Y}_iJ_i\bar{K}_i$, which are obtained *w.r.t.* y_i .The resulting formula is absorptive (*i.e.*, it has no term that can absorb another [20,38]), and hence it represents the complete sum of f as

$$CS(f) = Y_i(\bar{J}_i\bar{y}_i \vee K_i y_i \vee \bar{J}_i K_i) \vee \bar{Y}_i(J_i\bar{y}_i \vee \bar{K}_i y_i \vee J_i\bar{K}_i) = 0. \tag{6}$$

The expression in (6) is a disjunction of six terms equated to zero. This is exactly equivalent to each of the terms in (6) being individually equated to zero [20,38]. These six equations (See Table 2) constitute all the propositions that can be stated about the JK flip flop [32]. However, we might use the equivalence [20,38]:

$$\{A \to B\} \equiv \{A\bar{B} = 0\}, \tag{7}$$

to convert each of the six equational statements in Table 2 to any of eight equivalent implicational forms, as shown in Table 2.

Complete information about the JK flip flop is possible by selecting one of the nine equivalent statements from each of the six main rows in Table 2. Though Table 2 provides a wealth of propositions about the JK flip flops, many of these propositions are redundant as they are deducible from others. In fact, only four independent statements or propositions suffice to characterize the JK flip flop.

By the "independence" requirement above we rule out the following cases:

a) Selection of the first three successive equations $Y_i\bar{J}_i\bar{y}_i = 0$, $Y_iK_iy_i = 0$, and $Y_i\bar{J}_iK_i = 0$ or the next three consecutive equations $\bar{Y}_iJ_i\bar{y}_i = 0$, $\bar{Y}_i\bar{K}_iy_i = 0$, and $\bar{Y}_iJ_i\bar{K}_i = 0$. In each case, the third equation is simply deducible from the former two through a consensus operation.
b) Selection of two statements that belong to the same major row, since the nine statements in the same major row are simply equivalent.
c) Selection of three statements that belong to the first three major rows or to the last three major rows.

The simplest characterization is naturally the characterization *via* the equational forms in major (double) rows 1, 2, 4, and 5, (highlighted in Table 2). These equations are deducible from the original characteristic function (5) equated to zero, and they are neutral about utility to analysis or synthesis. By contrast, consider the following four implicational statements

$$Y_i\bar{y}_i \to J_i \tag{8a}$$
$$Y_iy_i \to \bar{K}_i \tag{8b}$$
$$\bar{Y}_i\bar{y}_i \to \bar{J}_i \tag{8c}$$
$$\bar{Y}_iy_i \to K_i. \tag{8d}$$

which are again selected from major rows 1, 2, 4, and 5. The implicational statements(8) are deliberately chosen to have the present state y_i and next state Y_i in the antecedents of the implications, and have either excitation J_i or excitation K_i in the consequents. This selection is not neutral but it is synthesis oriented. In fact, using techniques of Boolean reasoning [20,38], equations (8) can be seen as a precise translation of the excitation table (Table 1(c)). For example, the implication $(Y_i\bar{y}_i \to J_i)$ in (8a) is interpreted via the *Principle of Assertion* [20] to be precisely equivalent to the second line in Table (1(c)), which can be read as

$$\{y_i = 0, Y_i = 1\} \to \{J_i = 1, K_i = d\}. \tag{9}$$

Note that this implication keeps silent about K_i. which is its appropriate way of saying that K_i is a don't care.

A third selection for a characterizing set (also taken from major rows 1, 2, 4, and 5) is

$$\bar{J}_i \to \bar{Y}_i \vee y_i, \tag{10a}$$
$$K_i \to \bar{Y}_i \vee \bar{y}_i, \tag{10b}$$
$$J_i \to Y_i \vee y_i, \tag{10c}$$
$$\bar{K}_i \to Y_i \vee \bar{y}_i. \tag{10d}$$

Table 2. All possible statements that can be made about JK flip flops (arranged in six major (double) rows)

Equational form	Implicational form			
$\boldsymbol{Y_i\bar{J}_i\bar{y}_i = 0}$	$Y_i\bar{J}_i\bar{y}_i \to 0$	$Y_i\bar{J}_i \to y_i$	$Y_i\bar{y}_i \to J_i$	$\bar{J}_i\bar{y}_i \to \bar{Y}_i$
	$Y_i \to J_i \vee y_i$	$\bar{J}_i \to \bar{Y}_i \vee y_i$	$\bar{y}_i \to \bar{Y}_i \vee J_i$	$1 \to \bar{Y}_i \vee J_i \vee y_i$
$\boldsymbol{Y_iK_iy_i = 0}$	$Y_iK_iy_i \to 0$	$Y_iK_i \to \bar{y}_i$	$Y_iy_i \to \bar{K}_i$	$K_iy_i \to \bar{Y}_i$
	$Y_i \to \bar{K}_i \vee \bar{y}_i$	$K_i \to \bar{Y}_i \vee \bar{y}_i$	$y_i \to \bar{Y}_i \vee \bar{K}_i$	$1 \to \bar{Y}_i \vee \bar{K}_i \vee \bar{y}_i$
$\boldsymbol{Y_i\bar{J}_iK_i = 0}$	$Y_i\bar{J}_iK_i \to 0$	$Y_i\bar{J}_i \to \bar{K}_i$	$Y_iK_i \to J_i$	$\bar{J}_iK_i \to \bar{Y}_i$
	$Y_i \to J_i \vee \bar{K}_i$	$\bar{J}_i \to \bar{Y}_i \vee \bar{K}_i$	$K_i \to \bar{Y}_i \vee J_i$	$1 \to \bar{Y}_i \vee J_i \vee \bar{K}_i$
$\boldsymbol{\bar{Y}_iJ_i\bar{y}_i = 0}$	$\bar{Y}_iJ_i\bar{y}_i \to 0$	$\bar{Y}_iJ_i \to y_i$	$\bar{Y}_i\bar{y}_i \to \bar{J}_i$	$J_i\bar{y}_i \to Y_i$
	$\bar{Y}_i \to \bar{J}_i \vee y_i$	$J_i \to Y_i \vee y_i$	$\bar{y}_i \to Y_i \vee \bar{J}_i$	$1 \to Y_i \vee \bar{J}_i \vee y_i$
$\boldsymbol{\bar{Y}_i\bar{K}_iy_i = 0}$	$\bar{Y}_i\bar{K}_iy_i \to 0$	$\bar{Y}_i\bar{K}_i \to \bar{y}_i$	$\bar{Y}_iy_i \to K_i$	$\bar{K}_iy_i \to Y_i$
	$\bar{Y}_i \to K_i \vee \bar{y}_i$	$\bar{K}_i \to Y_i \vee \bar{y}_i$	$y_i \to Y_i \vee K_i$	$1 \to Y_i \vee K_i \vee \bar{y}_i$
$\boldsymbol{\bar{Y}_iJ_i\bar{K}_i = 0}$	$\bar{Y}_iJ_i\bar{K}_i \to 0$	$\bar{Y}_iJ_i \to K_i$	$\bar{Y}_i\bar{K}_i \to \bar{J}_i$	$J_i\bar{K}_i \to Y_i$
	$\bar{Y}_i \to \bar{J}_i \vee K_i$	$J_i \to Y_i \vee K_i$	$\bar{K}_i \to Y_i \vee \bar{J}_i$	$1 \to Y_i \vee \bar{J}_i \vee K_i$

The implications in (10) are the converses of those in (8) and hence they have antecedents involving either J_i or K_i and consequents involving both y_i and Y_i. The selection in (10) is analysis oriented. In fact, this selection is precisely equivalent to the next-state table (Table 1(a) and Table 1(b)). For example, the statement in (10a) is equivalent to the first four lines in Table 1(a),combined together, since they collectively state

$$\bar{J}_i \to \bar{y}_i\bar{Y}_i \vee y_iY_i \vee \bar{y}_i\bar{Y}_i \vee y_i\bar{Y}_i = \bar{Y}_i \vee y_i. \qquad (11a)$$

It is also equivalent to the first two lines of Table 1(c), combined together, since they collectively state

$$\bar{J}_i \to (y_i \odot Y_i) \vee \bar{Y}_i = y_iY_i \vee \bar{y}_i\bar{Y}_i \vee \bar{Y}_i = \bar{Y}_i \vee y_i. \, (11b)$$

where the last equality is obtained by absorbing the subsuming term $\bar{y}_i\bar{Y}_i$ in the subsumed term $\bar{Y}_i$ and enforcing the reflection law $(y_iY_i \vee \bar{Y}_i) = (y_i \vee \bar{Y}_i)$.

4. BOOLEAN-EQUATION SOLVING FOR EXCITATIONS

In this section, we employ methods of Boolean-equation solving [39-44] to obtain all possible solutions for the excitations in terms of the present state and next state. We consider two cases. In the first case, we concentrate on a single flip flop i and seek solutions of its excitations J_i and K_i in terms of its present state y_i and next state Y_i. In the second case, we explicitly acknowledge that Y_i is not an independent variable and recognize that it is a function of present states of all flip flops (as well as input variables).

4.1 Study of a Single Flip Flop Individually

Consider a single JK flip flop numbered i whose characteristic equation (2) is repeated herein for convenience, *i.e.*,

$$Y_i = J_i\bar{y}_i \vee \bar{K}_iy_i. \qquad (2)$$

To solve for J_i and K_i in terms of Y_i and y_i, we first convert (2) to the form of a single equation of a function equated to 1,*i.e.,* to the complement of (4), namely

$$g(y_i.Y_i;\ J_i.K_i) = 1, \qquad (12)$$

where g is the complement of f in (5), and is given by

$$\begin{aligned} g &= Y_i \odot (J_i\bar{y}_i \vee \bar{K}_iy_i) \\ &= Y_i(J_i\bar{y}_i \vee \bar{K}_iy_i) \\ &\quad \vee \bar{Y}_i(\bar{J}_i\bar{y}_i \vee K_iy_i) \\ &= \bar{J}_i(\bar{Y}_i\bar{y}_i) \vee K_i(\bar{Y}_iy_i) \vee J_i(Y_i\bar{y}_i) \vee \bar{K}_i(Y_iy_i) \\ &= (\bar{J}_i)\bar{Y}_i\bar{y}_i \vee (K_i)\bar{Y}_iy_i \vee (J_i)Y_i\bar{y}_i \\ &\quad \vee (\bar{K}_i)Y_iy_i. \end{aligned} \qquad (13)$$

The function g in (13) can be viewed as $g(J_i, K_i)$ where$g: B^2 \to B$, and$B = B_{16} = FB(Y_i.y_i)$ is the free Boolean algebra with 2 generates $Y_i and y_i$, $2^2 = 4$ atoms given by $\bar{Y}_i\bar{y}_i, \bar{Y}_iy_i, Y_i\bar{y}_i and Y_iy_i$, and $2^4 = 16$ elements. These elements can be identified as all the switching functions of the two variables $Y_i and y_i$. Fig. 2(a) is the natural (also called variable-entered) map for $g(J_i.K_i)$. Each of the four atoms of the underlying Boolean algebra is entered twice in this map. Hence the number of particular solutions for (12) is $2 * 2 * 2 * 2 = 16$ [11,43,44].

Fig. 2(a) can be used to construct the auxiliary function $G(J_i, K_i, \boldsymbol{p})$ in Fig. 2(b). Since each of the four atoms $\bar{Y}_i\bar{y}_i, \bar{Y}_iy_i. Y_i\bar{y}_i and Y_iy_i$ appears twice in Fig. 2(a), each of the two appearances of each atom j is appended in Fig. 2(b) by a distinguishing binary tag selected from the orthonormal set $\{\bar{p}_{ij}. p_{ij}\}$ [43,44]. For example, Fig. 2(b) shows the atom $\bar{Y}_i\bar{y}_i$ appended (ANDed with) $\bar{p}_{i0}$ in the cell $\bar{J}_i\bar{K}_i$ and appended with p_{i0} in the cell $\bar{J}_iK_i$. Finally the solution for $J_i and K_i$ is written as [11,43,44]

$$\begin{aligned} J_i &= Disjunction\ of\ entries\ of\ the\ domain\ J_i\ in\ Fig. 2(b) = Y_i\bar{y}_i\bar{p}_{i2} \vee Y_iy_ip_{i3} \vee Y_i\bar{y}_ip_{i2} \vee \bar{Y}_iy_ip_{i1} \\ &= Y_i\bar{y}_i \vee Y_iy_ip_{i3} \vee \bar{Y}_iy_ip_{i1}. \end{aligned} \qquad (14a)$$

$$\begin{aligned} K_i &= Disjunction\ of\ entries\ of\ the\ domain\ K\ in\ Fig. 2(b) = \bar{Y}_i\bar{y}_ip_{i0} \vee \bar{Y}_iy_i\bar{p}_{i1} \vee Y_i\bar{y}_ip_{i2} \vee \bar{Y}_iy_ip_{i1} \\ &= \bar{Y}_iy_i \vee \bar{Y}_i\bar{y}_ip_{i0} \vee Y\bar{y}_ip_{i2}. \end{aligned} \qquad (14b)$$

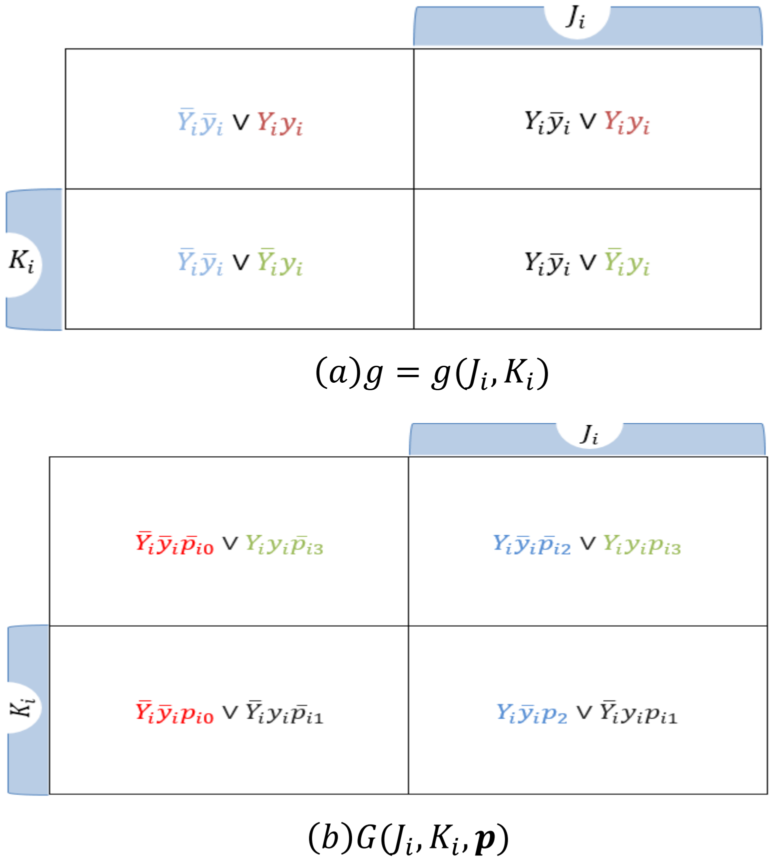


Fig. 2. Natural map for the function $g(J_i, K_i)$ and for the auxiliary function $G(J_i,\ K_i, \boldsymbol{p})$.

Equations (14) are a parametric solution for $J_i and K_i$, where each of the four independent parameters $p_{i0}, p_{i1}, p_{i2} and p_{i3}$ belongs to $B_2 = \{0.1\}, and$hence (14) can be used to deduce the 16 particular solutions of (12). We might elect to replace each of the four binary parameters by the same single parameter p_i (provided we let this parameter p_i belong to the underlying Boolean algebra $(B_{16} = FB(Y_i, y_i)))$. In this case, the solution becomes

$$J_i = Y_i\bar{y}_i \vee y_i p_i, \qquad (15a)$$
$$K_i = \bar{Y}_i y_i \vee \bar{y}_i p_i. \qquad (15b)$$

Equations (15) are another parametric solution for (12) that uses a single parameter p_i belonging to $B_{16} = FB(Y_i, y_i)$. This solution was obtained earlier by Brown [16]. The solutions (14) and (15) are equivalent, since they produce the same set of 16 particular solutions. Since none of the four atoms is absent in Fig. 2, the consistency condition for the equation $\{g = 1\}$ is satisfied trivially as an identity $\{0 = 0\}$.

An alternative way to solve for $J_i and K_i$ is to use the concept of atomic decomposition. Figs. 3(a) and 3(b) present the atomic decompositions of the variables $J_i and K_i, namely$

$$J_i = (J_{i0})\bar{Y}_i\bar{y}_i \vee (J_{i1})\bar{Y}_i y_i \vee (J_{i2})Y_i\bar{y}_i \vee (J_{i3})Y_i y_i, \qquad (16a)$$

$$K_i = (K_{i0})\bar{Y}_i\bar{y}_i \vee (K_{i1})\bar{Y}_i y_i \vee (K_{i2})Y_i\bar{y}_i \vee (K_{i3})Y_i y_i. \qquad (16b)$$

where the four atomic components of $J(J_{i0},\ J_{i1},\ J_{i2},\ J_3)$ and those of $K(K_{i0},\ K_{i1},\ K_{i2},\ K_{i3})$ are arbitrary binary values.

Substituting these decompositions into (13) we obtain the atomic decomposition of $g(J_i, K_i)$ as

$$
\begin{aligned}
g(J_i.K_i) &= (\bar{J})\bar{Y}_i\bar{y}_i \vee (K)\bar{Y}_i y_i \vee (J)Y_i\bar{y}_i \vee (\bar{K})Y_i y_i \\
&= \big((\bar{J}_0)\bar{Y}_i\bar{y}_i \vee (\bar{J}_1)\bar{Y}_i y_i \vee (\bar{J}_2)Y_i\bar{y}_i \vee (\bar{J}_3)Y_i y_i\big)\bar{Y}_i\bar{y}_i \\
&\vee \big((K_0)\bar{Y}_i\bar{y}_i \vee (K_1)\bar{Y}_i y_i \vee (K_2)Y_i\bar{y}_i \vee (K_3)Y_i y_i\big)\bar{Y}_i y_i \\
&\vee \big((J_0)\bar{Y}_i\bar{y}_i \vee (J_1)\bar{Y}_i y_i \vee (J_2)Y_i\bar{y}_i \vee (J_3)Y_i y_i\big)Y_i\bar{y}_i \\
&\vee \big((\bar{K}_0)\bar{Y}_i\bar{y}_i \vee (\bar{K}_1)\bar{Y}_i y_i \vee (\bar{K}_2)Y_i\bar{y}_i \vee (\bar{K}_3)Y_i y_i\big)Y_i y_i \\
&= (\bar{J}_0)\bar{Y}_i\bar{y}_i \vee (K_1)\bar{Y}_i y_i \vee (J_2)Y_i\bar{y}_i \vee (\bar{K}_3)Y_i y_i \qquad (17)
\end{aligned}
$$

Fig. 3(c) illustrates the atomic decomposition of $g(J_i, K_i)$ as given by (17). The equation to be solved forces each individual entry in the map of Fig. 3(c) to be 1. Hence, we obtain

$$J_{i0} = 0, \qquad (18a)$$
$$K_{i1} = 1, \qquad (18b)$$
$$J_{i2} = 1, \qquad (18c)$$
$$K_{i3} = 0. \qquad (18d)$$

Fig. 4 shows the general solution of $J_i and K_i$ in terms of $Y_i and y_i$ in the atomic-decomposition form. Each of the atomic components J_{i1}, J_{i3}, K_{i0} and K_{i2} is still unspecified or arbitrary, and can be assigned a value of 0 or 1 independently of each of the other components. The four possible assignments for the two constants J_{i1} and J_{i3} are shown in Fig. 5, together with the corresponding four possible solutions forJ_i.Likewise, the four possible assignments for the two constants K_{i0} and K_{i3} are also shown in Fig. 5, together with the corresponding four possible solutions for K_i. Any of the four solutions of J_i can go with any of the four solutions for K_i, and hence, we have a totality of $4 * 4 = 16$ particular solutions, as expected.

We can easily verify that each of the four solutions for J_i satisfies

$$J_i\bar{y}_i = Y_i\bar{y}_i, \qquad (19)$$

while each of the four solutions for K_i satisfies

$$\bar{K}_i y_i = Y_i y_i, \qquad (20)$$

and the conditions (19 and 20) can be ORed to obtain the original equation (2), namely

$$J_i\bar{y}_i \vee \bar{K}_i y_i = Y_i(\bar{y}_i \vee y_i) = Y_i. \qquad (2a)$$

For convenience, we redraw the solution in Fig. 4 as shown in Fig. 6 with variable Y_i being transferred from its original role as a map variable so as to become an entered variable. Fig. 6 involves two Boolean quotients (See Appendix A) and two don't care values.

The essence of Fig. 6 was earlier obtained by Brown [16], as he eliminated the next state Y_i from his solution (15) by invoking (A.5) to obtain

$$J_i = (Y_i/\bar{y}_i)\bar{y}_i \vee y_i p_i, \qquad (21a)$$
$$K_i = (\bar{Y}_i/y_i)y \vee \bar{y}_i p_i. \qquad (21b)$$

The parametric equations above might be rewritten in the following don't-care notation [39-41,45]

$$J_i = (Y_i/\bar{y}_i)\bar{y}_i \vee d(y_i), \qquad (22a)$$
$$K_i = (\bar{Y}_i/y_i)y_i \vee d(\bar{y}_i). \qquad (22b)$$

or in the equivalent double-inequality notation

$$(Y_i/\bar{y}_i)\bar{y}_i \le J_i \le (Y_i/\bar{y}_i)\bar{y}_i \vee y_i, \qquad (23a)$$
$$(\bar{Y}_i/y_i)y_i \le K_i \le (\bar{Y}_i/y_i)y_i \vee \bar{y}_i. \qquad (23b)$$

An important word of caution is due here. While the entries of the maps in Fig. 2 are binary values that are elements of $B_2 = \{0.1\}$, the symbols d_J and d_K in Fig. 6 are functions of all circuit variables other than y_i itself.

4.2 Design of a 4-State 1-Input Synchronous Sequential Circuit

To demonstrate and clarify the results of the former subsection, we consider the problem of design of a 4-state single-input synchronous sequential circuit. We assume that the circuit is specified by the double map for the two next-state variables shown in Fig. 7(a). The circuit is synthesized with two JK flip flops whose excitations are given by the four maps in Figs. 7(b)-7(e), respectively. Each of these maps inherits half its entries from the double map in Fig. 7(a), while the rest of its entries are don't cares. Contrary to common practice (which would depict each of the don't cares in Figs. 7(b)-7(e) simply as 'd'), we deliberately depict each don't care by a distinguishing label, to stress that it can be arbitrarily assigned a $0/1$ value independently of the other don't cares. Each of the pair of Figs. (7a) and (7b) and the pair of Figs. (7c) and (7d) is an expanded detailed version of Fig. 6. These expanded figures demonstrate clearly how a Boolean quotient is graphically constructed and confirm that the symbols d_J and d_K in Fig. 6 are functions rather than single values.

		Y_i
	J_{i0}	J_{i2}
y_i	J_{i1}	J_{i3}

(a) J_i

		Y_i
	K_{i0}	K_{i2}
y_i	K_{i1}	K_{i3}

(b) K_i

		Y_i
	$\bar{J}_{i0}$	J_{i2}
y_i	K_{i1}	$\bar{K}_{i3}$

(c) $g = g(J_i, K_i)$

Fig. 3. Atomic decompositions of the variables J_i and K_i as well as the function $g(J_i, K_i)$

		Y_i
	0	1
y_i	J_{i1}	J_{i3}

J_i

		Y_i
	K_{i0}	K_{i2}
y_i	1	0

K_i

Fig. 4. The general solution for J_i and K_i in terms of Y_i and y_i. The atomic components J_{i1}, J_{i3}, K_{i0} and K_{i2} are not specified and remain to be arbitrary binary constants

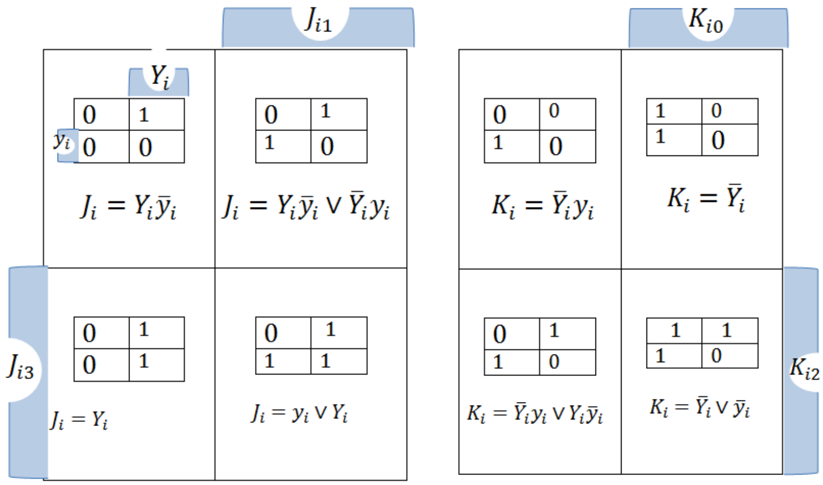


Fig. 5. Demonstration that each of the four possible solutions for $J_i$can be combined with each of the four possible solutions for K_i to generate sixteen solutions for (J_i, K_i)

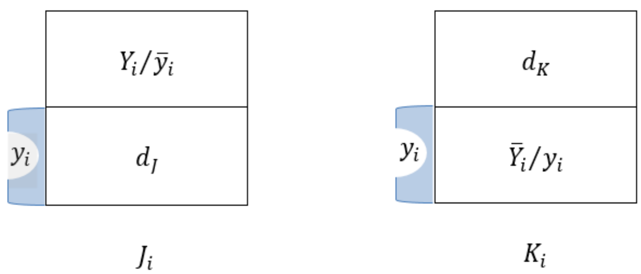


Fig. 6. The maps in Fig. 4 redrawn to make the variable Y_i an entered variable

5. FULL DETERMINATION OF THE JK EXCITATIONS THROUGH APPROPRIATE ASSIGNMENTS TO DON'T-CARES

So far, we have obtained the excitations $J_i and K_i$ as incompletely-specified functions (partially defined functions). Table 3(a) shows some possible assignments for the function d_J, each of which leads to complete specification of the excitation J_i. Likewise, Table 3(b) shows some possible assignments for the function d_K, each of which leads to complete specification of the excitation K_i. Any of the five assignments in Table 3(a) can go with any of the five assignments in Table 3(b). The two tables (combined) identify 25 possible complete specifications of the excitations of the JK flip flop. A few observations are in order:

- If we use the first assignment in each of Table 3(a) and Table 3(b), we obtain the most compact formulas for $J_i and\ K_i$, which are independent of y_i. These are the formulas of choice when synthesizing synchronous sequential circuits with JK flip flops.
- Using the second assignment in each table, we obtain $J_i = \bar{K}_i = Y_i$ (a value that we call D_i), and the JK flip flop reduces to a D flip flop.
- Using the third assignment in each table, we obtain $J_i = K_i = Y_i \oplus y_i$(a value that we call T_i), and the JK flip flop reduces to a T flip flop.
- The JK flip flop can be used as an SR flip flop provided $J_i K_i = 0$. This is the case when using the second or fourth assignment in each of Table 3(a) and Table 3(b).

6. COMPARISON OF FAMOUS TYPES OF FLIP FLOPS

We devote this section to a brief logical comparison of six types of flip flops, namely, the $D,\ T,\ SR,\ JK,\ RST,\ and FIVEX$ flip flops. Since a flip flop is named by upper-case letters depicting its inputs, these flip flops possess $1, 1, 2, 2, 3, and\ 5$ inputs, respectively. The SR and RST flip flops have certain constraints on their excitations, while the remaining flip flops do not. The characteristic equation (together with any constraints) for each of these flip flops is shown in entered-map form in Fig. 8. Only three types of the flip flops considered (JK, RST, $andFIVEX$) have the full flexibility of assigning to Y_i any of four functions $\{0, 1, y_i, \bar{y}_i\}$ of the single variable y_i. The JK flip flop is distinguished to have this flexibility with just two inputs. Table 4 restates the aforementioned characteristic equations (with constraints, if any) in algebraic form. Table 4 also presents same of the possible excitation equations for each flip flop. To complete the picture, Table 5 shows how to convert any of the six types of flip flops to any of the first four types. The conversion equations are such that if one substitutes a conversion equation in the characteristic equation of the original flip flop, then the characteristic equation of the new flip flop is obtained. Table 5 is intended as a demonstration of the functionalities of the various types of flip flops, and not as a recommendation for practical application.

Table 3(a). Some potential assignments for the function d_J so as to specify the excitation J_i completely

Assignment number	d_J	$J_i = (Y_i/\bar{y}_i)\bar{y}_i \vee d_J y_i$
1	$Y_i/\bar{y}_i$	$(Y_i/\bar{y}_i)\bar{y}_i \vee (Y_i/\bar{y}_i)y_i = (Y_i/\bar{y}_i)(\bar{y}_i \vee y_i) = Y_i/\bar{y}_i$
2	Y_i/y_i	$(Y_i/\bar{y}_i)\bar{y}_i \vee (Y_i/y_i)y_i = Y_i\bar{y}_i \vee Y_i y_i = Y_i(\bar{y}_i \vee y_i) = Y_i$
3	$\bar{Y}_i/y_i$	$(Y_i/\bar{y}_i)\bar{y}_i \vee (\bar{Y}_i/y_i)y_i = Y_i\bar{y}_i \vee \bar{Y}_i y_i = Y_i \oplus y_i$
4	0	$(Y_i/\bar{y}_i)\bar{y}_i = Y_i\bar{y}_i$
5	1	$(Y_i/\bar{y}_i)\bar{y}_i \vee y_i = Y_i\bar{y}_i \vee y_i = Y_i \vee y_i$

Table 3(b). Some potential assignments for the function d_K so as to specify the excitation K_i completely

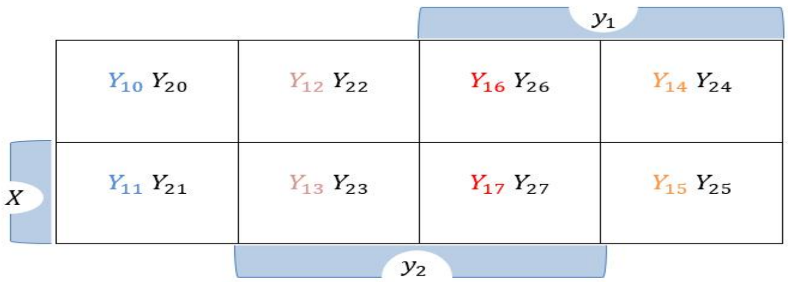
Assignment Number	d	$K_i = (\bar{Y}_i/y_i)y_i \vee d_k\bar{y}_i$
1	$\bar{Y}_i/y_i$	$(\bar{Y}_i/y_i)y_i \vee (\bar{Y}_i/y_i)\bar{y}_i = (\bar{Y}_i/y_i)(y_i \vee \bar{y}_i) = \bar{Y}_i/y_i$
2	$\bar{Y}_i/\bar{y}_i$	$(\bar{Y}_i/y_i)y_i \vee (\bar{Y}_i/\bar{y}_i)\bar{y}_i = \bar{Y}_i y_i \vee \bar{Y}_i\bar{y}_i = \bar{Y}_i(y_i \vee \bar{y}_i) = \bar{Y}_i$
3	$Y_i/\bar{y}_i$	$(\bar{Y}_i/y_i)y_i \vee (Y_i/\bar{y}_i)\bar{y}_i = \bar{Y}_i y_i \vee Y_i\bar{y}_i = Y_i \oplus y_i$
4	0	$(\bar{Y}_i/y_i)y_i = \bar{Y}_i y_i$
5	1	$(\bar{Y}_i/y_i)y_i \vee \bar{y}_i = \bar{Y}_i y_i \vee \bar{y}_i = \bar{Y}_i \vee \bar{y}_i$

Table 4. Analysis and design equations for six famous types of flip flops

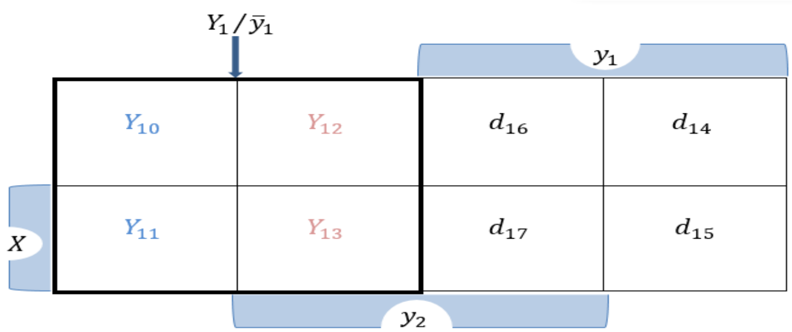
	Analysis Equation (Formula for next state)	**Design Equations Formula(s) for excitation(s)**	
D	$Y_i = D_i$	$D_i = Y_i$	
T	$Y_i = T_i \oplus y_i$	$T_i = Y_i \oplus y_i$	
SR	$Y_i = S_i \vee \bar{R}_i y_i, \quad S_i R_i = 0$	$S_i = Y_i$ $R_i = \bar{Y}_i$	$S_i = Y_i\bar{y}_i$ $R_i = \bar{Y}_i y_i$
JK	$Y_i = J_i\bar{y}_i \vee \bar{K}_i y_i$	$J_i = Y_i/\bar{y}_i$ $K_i = \bar{Y}_i/y_i$	See Section 5
RST (denoted by lower-case inputs)	$Y_i = s_i \vee t_i\bar{y}_i \vee \bar{r}_i\bar{t}_i y_i$ $= s_i\bar{t}_i\bar{y}_i \vee \bar{r}_i t_i\bar{y}_i \vee \bar{r}_i\bar{t}_i y_i,$ $r_i s_i = r_i t_i = s_i t_i = 0$	$s_i = Y_i$ $r_i = \bar{Y}_i$ $t_i = 0$	$s_i = 0$ $r_i = 0$ $t_i = Y_i \oplus y_i$
$FIVEX$	$Y_i = \bar{y}_i\bar{X}_i F_i \vee \bar{y}_i X_i I_i \vee y_i\bar{X}_i V_i$ $\vee y_i X_i E_i$	$F_i = Y_i$ $I_i = Y_i$ $V_i = Y_i$ $K_i = Y_i$ $X_i\ free$	

Table 5. Conversion of a flip-flop type to another

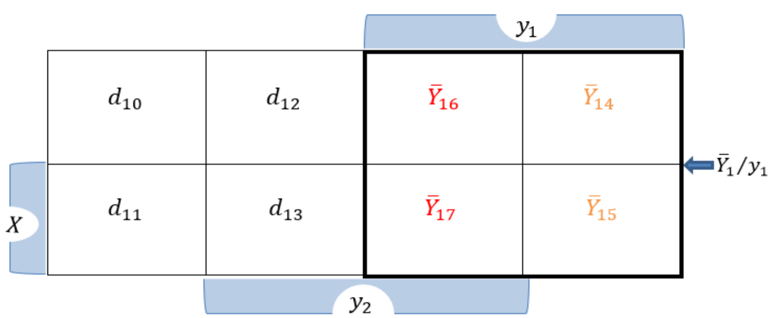
To \ From	D	T	SR	JK	RST (denoted herein as inputs $r_i s_i t_i$)		$FIVEX$
D	–	$T_i = D_i \oplus y_i$	$S_i = D_i$ $R_i = \overline{D}_i$	$J_i = D_i$ $K_i = \overline{D}_i$	$r_i = \overline{D}_i$ $s_i = D_i$ $t_i = 0$	$r_i = 0$ $s_i = 0$ $t_i = D_i \oplus y_i$	$F_i = D_i$ $I_i = D_i$ $V_i = D_i$ $E_i = D_i$ $X_i = free$
T	$D_i = T_i \oplus y_i$	–	$S_i = T_i \bar{y}_i$ $R_i = T_i y_i$	$J_i = T_i$ $K_i = T_i$	$r_i = 0$ $s_i = 0$ $t_i = T_i$	$r_i = T_i y_i$ $s_i = T_i \bar{y}_i$ $t_i = 0$	$F_i = T_i$ $I_i = T_i$ $V_i = \overline{T}_i$ $E_i = \overline{T}_i$ $X_i = free$
SR $(S_i R_i = 0)$	$D_i = S_i \vee \bar{R}_i y_i$	$T_i = S_i \bar{y}_i \vee R_i y_i.$	–	$J_i = S_i$ $K_i = R_i$	$r_i = R_i$ $s_i = S_i$ $t_i = 0$	$r_i = 0$ $s_i = 0$ $t_i = S_i \bar{y}_i \vee R_i y_i$	$F_i = S_i$ $I_i = S_i$ $V_i = \bar{R}_i$ $E_i = \bar{R}_i$ $X_i = free$
JK	$D_i = J_i \bar{y}_i \vee \overline{K}_i y_i$	$T_i = J_i \bar{y}_i \vee K_i y_i$	$S_i = J_i \bar{y}_i$ $R_i = K_i y_i$	–	$r_i = K_i y_i$ $s_i = J_i \bar{y}_i$ $t_i = 0$	$r_i = 0$ $s_i = 0$ $t_i = J_i \bar{y}_i \vee K_i y_i$	$F_i = J_i$ $I_i = J_i$ $V_i = \overline{K}_i$ $E_i = \overline{K}_i$ $X_i = free$



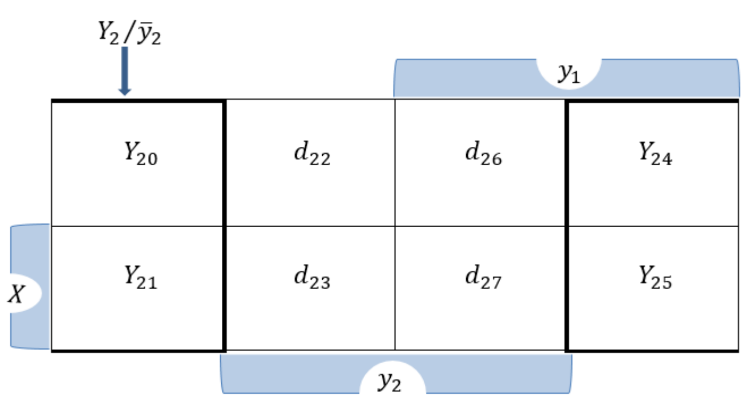
$(a) Y_1 Y_2$



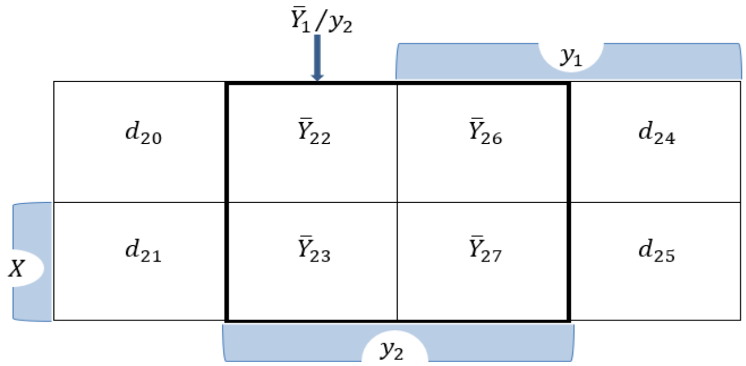
$(b) J_1$



$(c)\ K_1$

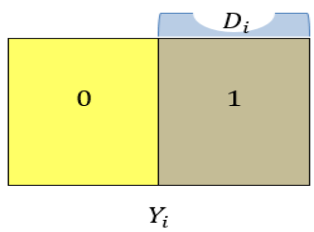


$(d) J_2$

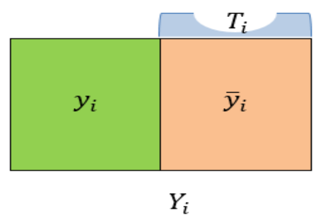


$(e)\ K_2$

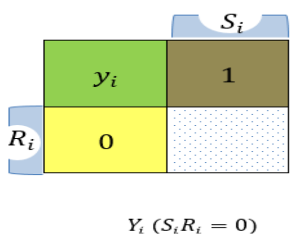
Fig. 7. A general double map for the two next-state variables y_1 and y_2 of a 4-state synchronous sequential circuit with a single input X, and the corresponding maps for the excitations of two JK flip flops that synthesize the circuit



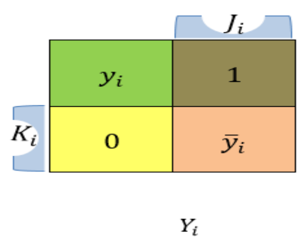
$(a)\ The\ D\ flip\ flop$



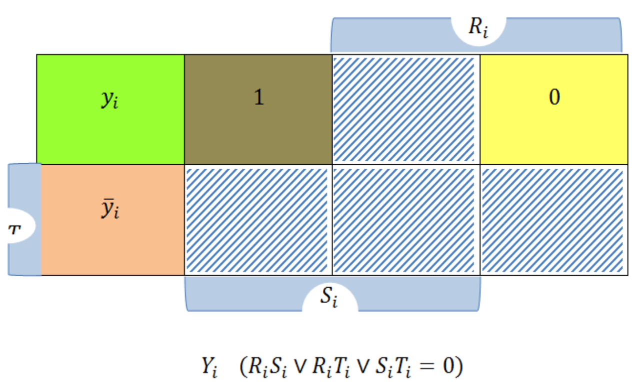
$(b)\ The\ T\ flip\ flop$



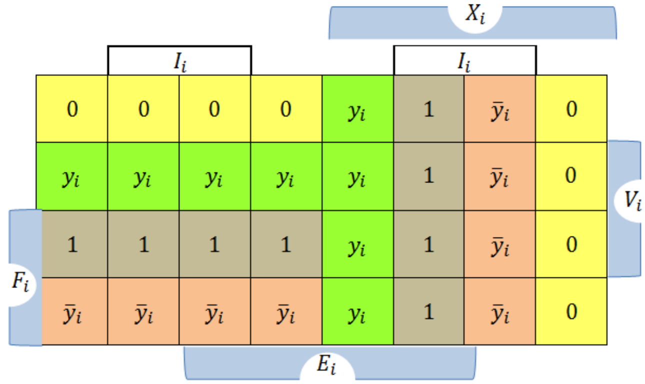
$(c)\ The\ SR\ flip\ flop$



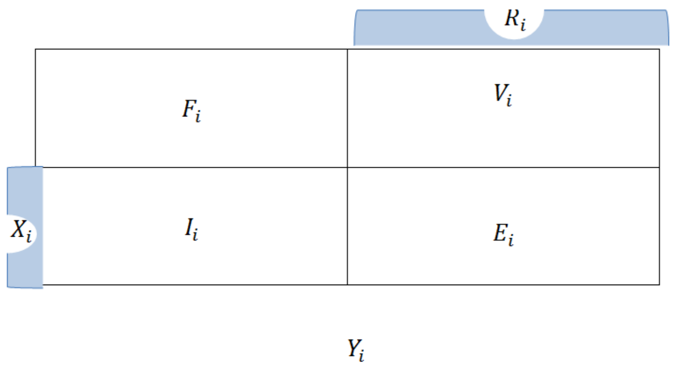
$(d) The\ JK\ flip\ flop$



(e) The RST flip flop



(f_1) *the FIVEX flip flop*



(f_2) *the FIVEX flip flop*

Fig. 8. Variable-entered next-state maps for various flip flops

A modern digital network, especially one that is based on large-scale integration devices (such as a Field-Programmable Gate Array (*FPGA*)) employs a different type of flip-flop called the DE flip flop (or simply the E flip flop), which is an extension of the D type [21,46-49]. This flip flop is similar to a standard D flip flop except that the D input is only enabled when the input E (for "Enable") is equal to logic 1. When the input E is equal to logic 0, the flip flop remains in its current state. Hence, the DE flip flop has the capacity to store an input value only upon request or enabling. This behavior differs from that of the D flip flop, which stores a new value (unconditionally) at each active edge of the clock. The characteristic equation for this flip flop might be written as

$$Y_i = y_i \bar{E}_i \vee D_i E_i. \qquad (24)$$

Detailed information about the E flip flop is available in [48], where it is given the designation E-PET with "PET" standing for "Positive-Edge-Triggered."

7. CONCLUSIONS

This paper is a tutorial exposition about flip flops, with a stress on a widely used 2-input type, namely, JK flip flops. The paper reviewed conventional methods (that are already known) for characterizing JK flip flops. Then it employed novel mathematical methods for further characterization of these flip flops. The methods used included ones of logic deduction, Boolean-equation solving, and don't-care assignment. The paper also includes a brief coverage of other related types of flip flops of various inputs and capabilities including the D flip flop, T flip flop, SR flip flop, RST flip flop, and $FIVEX$ flip flops. Specifically, we explored the relation of the JK flip flop to each of these flip flops. The immediate benefit gained from this paper is that it helps facilitate the analysis and synthesis of sequential digital circuits.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Appendix A. Boolean Quotients and the Boole-Shannon Expansion

The concept of a Boolean quotient is a switching-algebraic concept that can be conveniently used to facilitate Boolean manipulations. Given a two-valued Boolean function f and a term t, the Boolean quotient of f with respect to t, denoted by (f/t) or $(f|t)$, is defined to be the function formed from f by explicitly imposing the condition $\{t=1\}$ (See Brown [20],Rushdi and Rushdi [50], or Rushdi [51]), *i.e.,*

$$f/t = [f]_{t=1}, \tag{A.1}$$

The Boolean quotient is also known as a ratio [2], a subfunction [45,52] or a restriction [53]. An important feature of Boolean quotients is that the conjunction of a term with a function is equal to the conjunction of the term with the Boolean quotient of the function with respect to the term, *viz.,*

$$t \wedge f \;= t \;\wedge (f/t). \tag{A.2}$$

If the term t is implied by the function f (*i.e.*, $f \leq t$, $f \rightarrow t$, $f = t \wedge f$), then (A.2) reduces to

$$f \;= t \;\wedge (f/t). \tag{A.3}$$

The concept of the Boolean quotient has a striking similarity to that of conditional probability [54-58], but perhaps the most important utilization of the Boolean quotient is its use in the Boole-Shannon Expansion, which constitutes the most fundamental theorem of Boolean algebra (See Brown [20], Rushdi and Ghaleb [30], or Rushdi and Rushdi [50])

$$f(\boldsymbol{X}) = (\bar{X}_i \wedge (f(\boldsymbol{X})/\bar{X}_i)) \vee (X_i \wedge (f(\boldsymbol{X})/X_i)), \tag{A.4}$$

For example, the next state Y_i of flip flop number i can be expressed in terms of the present state y_i of the same flip flop as

$$Y_i = (Y_i/\bar{y}_i)y_i \vee (Y_i/y_i)y_i. \tag{A.5}$$

The two Boolean quotients in (A.5) are independent of the state of flip flop i. They are functions of other variables of the circuit, including inputs to the excitation logic and the present states of other flip flops.
The Boole-Shannon Expansion can be conveniently displayed in the map form of Fig. A.1 [30].

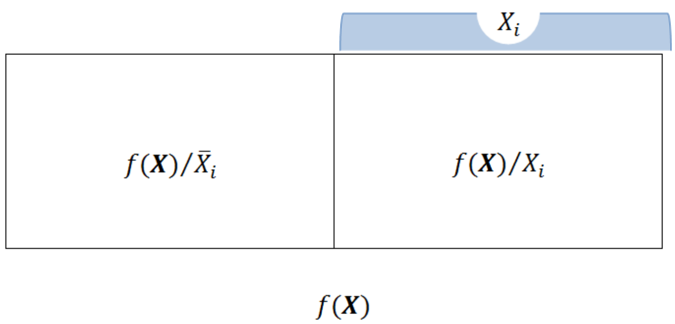

Fig. A.1. A variable-entered Karnaugh map illustrating the Boole-Shannon expansion (A.4)

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